The market price of mortality risk

Sonny Loo and Allen Truslove use modern finance theory to price Australian mortality liabilities

Customers buy insurance to avoid carrying the high variability inherent in the risk arising from high- value, infrequently occurring events. A known premium replaces the otherwise uncertain cost. Customers pay a margin over the average claim cost as the price of removal of the variability.

Market value of a portfolio of death risks

The market value of a portfolio of death risks is greater than the average claim cost. The excess over the average claim cost is the market price of the variability borne. How should the market value of a portfolio of death risks be calculated?

To put a value on the variability of a portfolio of death risks we must first quantify the variability. The variability has two sources:

- diversifiable variance, which as the number of lives increases reduces as a proportion of average claim cost;
- systemic variance, which arises from fluctuations in mortality rates across the whole population due to harsh winters, disease epidemics, etc.

For diversifiable variance the 'law of averages' (as Kolmogorov's Strong Law of Large Numbers is generally known) says that the experienced average claim cost tends towards the expected average claim cost as the number of claims increases. Hence, for large portfolios of death risks, it may be reasonable to neglect diversifiable variance because any variation is a very small fraction of the total.

Systemic mortality rate variance

To measure systemic mortality variance the mortality rates of the whole Australian population for the period 1901 to 1996 has been analysed. The actual mortality rates as compared with the trend line $q=0.093345-0.00004325xt$, for values of t from 1901 to 1996, is shown in figure 1. The variance measured relative to the trend line is shown in figure 2.

The Anderson-Darling normality test shows that the mortality rate fluctuations around the trend line are normally distributed with standard deviation $s=0.000569$, with test values of A^2 equal to 0.445 and p-value equal to 0.279. The normality test results are good. Note that the unbiased estimate of the standard deviation is $s=0.000572$.

For the empirical values of the mean and standard deviation, the beta distribution on a domain [0,1] very closely approximates the normal distribution. The normal distribution may be regarded as an approximation to the beta distribution here.

Suppose that the death rate is m and that systemic variation means that m is normally distributed with mean μ and standard deviation σ. Manipulation of moment-generating functions shows that for a portfolio of S lives the expected number of deaths is μS and the variance is $\mu S+\sigma^2S^2$. The variance per life insured is $\mu/S+\sigma^2$, which tends to σ^2 as S increases, as expected with diversification.

Figure 1

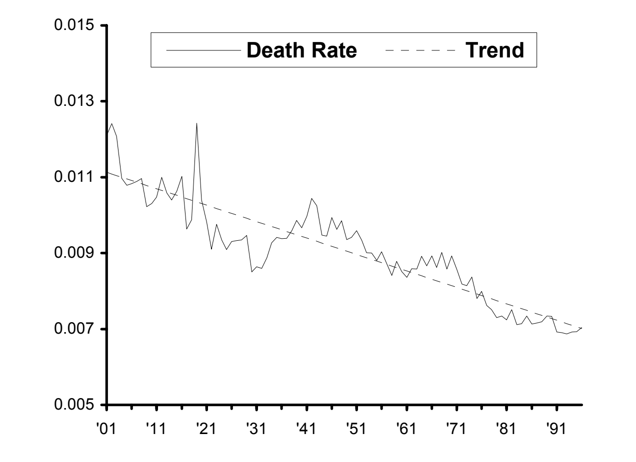
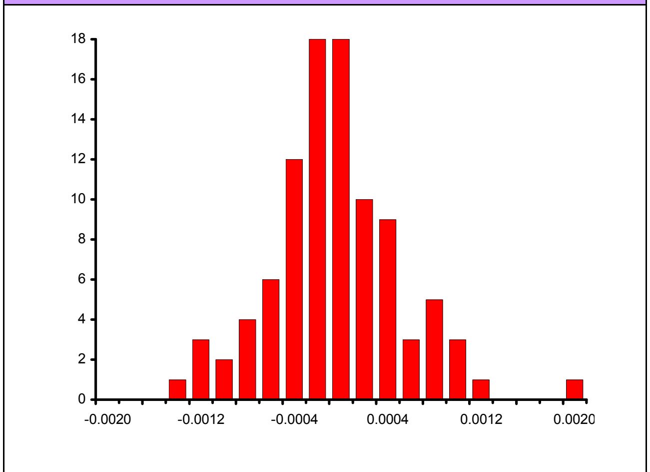


Figure 2



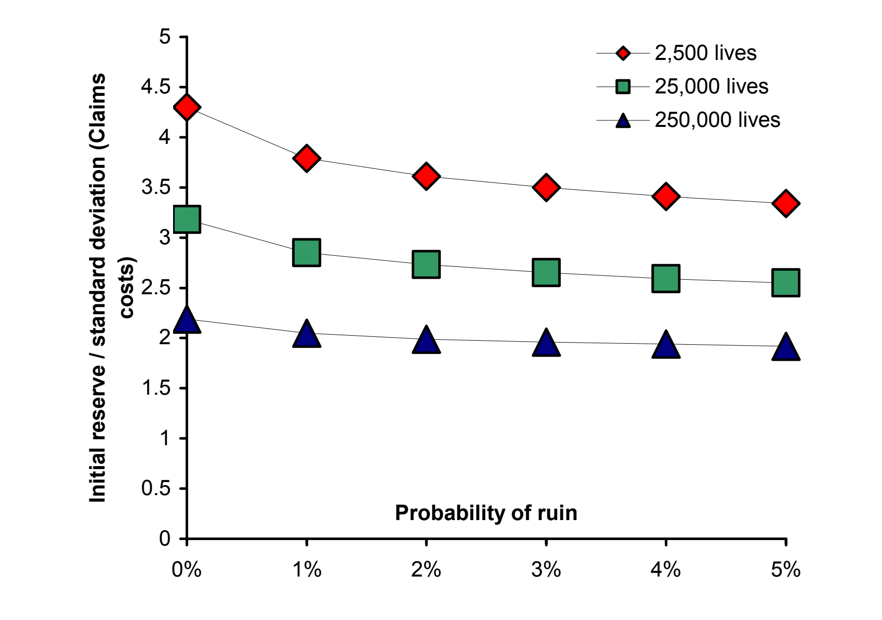
Market price of mortality variance

Since the mean μS is not equal to the variance $\mu S+\sigma^2S^2$ the process cannot be Poisson. The consequence is that Lundberg's ruin theory, which is based on the assumption of a Poisson process, cannot be applied to determine a margin for mortality fluctuation risk.

Modern financial theory calculates market values of both assets and liabilities using a risk-neutral equivalent martingale measure and a risk-free interest rate. Under a number of simplifying assumptions, this gives either the Black-Scholes option pricing model or a formula identical in form (but not in conceptual basis) to the Capital Asset Pricing Model.

Based on post-war experience, Australia's All Ordinaries Share Index implies a risk premium of 6.6% over government bond rates. This is a premium for a systemic standard deviation of 20%.

Figure 3



For Australia in 1998 the mean projected population mortality rate is 0.00693 so that the systemic standard deviation of 0.000569 is 8.2% of the mean. Since the price of systemic variance is proportional to its size, systemic mortality variance has an interest price of

$$(8.2\% / 20\%) \times 6.6\% = 2.7\%$$

relative to the whole population. This interest margin may be provided for by deduction from the risk-free rate to obtain the rate to be used for valuation purposes.

The absolute price of a given mortality variance is independent of the mean. That price is:

$$0.000569 \times (6.6\% / 20\%) = 0.000188.$$

The deduction from the interest rate is then:

$$(0.000188 / q) \times 100\%$$

for a mortality rate of q. For simplicity assume that the population mortality variance applies at all ages. A portfolio of insured lives has an average mortality rate of say 0.00150. The price expressed as an interest margin on a lower mean is then:

$$(0.000188 / 0.00150) \times 100\% = 12.5\%.$$

Prices for other mortality rates may be similarly derived. The method may be generalised to allow for variation in sums assured.

Interest rate for mortality liabilities

In general, if changes in the value of assets perfectly matched the change in value of liabilities then the position is risk-free. This position cannot be obtained for mortality variation because there is no suitable matching asset (other than reinsurance contracts).

Australian experience is that there is zero correlation between the stockmarket and mortality rates. The financial risk-free position is the investment in government bonds matching the expected pattern of payments. The yield on those investments gives the 'risk-free' rate in this case. The expected value of mortality liabilities is then the expected value of mortality payments, discounted at the 'risk-free' rate less the margin for mortality fluctuation derived above. The value thus derived is the market value.

The market value must not be confused with the 'funding value', which is the amount which if invested, together with the earnings expected to be earned on whatever type of asset is chosen, will fund the expected value of the liability.

Allowing for asset mismatching

If assets are invested in types of assets which introduce a financial mismatching risk, then reserves must be held to buffer the asset-mismatching risk. This is a gearing up of investment risk on the mortality claims reserve.

Let A be the amount of the asset fluctuation reserves. Let be L the amount of the mortality liability reserves calculated on the basis in the preceding section at an interest rate of i. Suppose that the investments actually held yield i', and that the required yield on A is j. Then $i'(A+L)=jA+iL$, so that:

$$i' = i' + (i' - i) \cdot (L/A).$$

This geared up rate is that required on assets A carrying the risk on both A & L.

> The price margin to cover mortality variance is set by the market

Fluctuation reserves

Mortality variance risk is carried by both the price margin received and by the initial reserve level. There is then a trade-off between the level of reserves and the probability of adequacy of those reserves. The market level of probability of adequacy is unknown.

Figure 3 shows, for a particular claim-size distribution, the reserve level (measured as a fraction of the standard deviation of claims), for a one year case and for a 12.5% margin, plotted against a range of ruin probabilities.

In practice the standard deviation of aggregate claims cost is a fraction of the average aggregate claims cost, so that the level of required reserves differs little with the probability of adequacy. Hence a suitable reserve level can be closely approximated.

Summary

- The margin or price received for carrying mortality fluctuation risk is determined by the systemic part of the mortality fluctuation risk.
- The market value of a portfolio of mortality risks is the sum of the average or expected claims payable, plus the price of the systemic part of the risk inherent in the claims variability, all discounted at a risk- free rate to allow for the time value of money.
- In practice suitable reserve levels for mortality fluctuation risk are not difficult to determine, because at high probabilities of adequacy the required reserve levels differ by little given the market-determined price margin for risk.
- The return on reserves is the price margin for mortality risk plus the investment return, perhaps geared up, received on assets.
- If assets and liabilities are mismatched to gear up the investment return on assets then reserves are also required to cover the increased asset risk.

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